Effective Shear Viscosity of Active Suspensions at Moderate Concentrations

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A number of recent experimental studies [1] have demonstrated that a suspension of self-propelled bacteria (*Bacillus subtilis*) at moderate concentrations may have an effective viscosity that is significantly (up to five to seven times) smaller than the viscosity of the ambient fluid. This is in sharp contrast to suspensions of hard passive inclusions, whose presence always increases viscosity. Detailed understanding of this viscosity phenomenon will help in the development of new materials and engineering solutions (e.g., improved micro-mixers). The 2D model developed in [2] captured an experimentally observed decrease of effective viscosity and provided an explanation for the underlying mechanisms.

The analysis was performed for moderate concentrations (about 9% by volume) of micro-swimmers that closely resemble the experimental settings, where a computer simulation is the only available tool. The analysis showed that the decrease in the effective viscosity observed in the physical experiments can be explained entirely from the point of view of hydrodynamic interactions. This is an important observation, since suspensions of bacteria represent a very complex system with a variety of phenomena occurring simultaneously, such as chemotaxis and secretion of proteins by living bacteria. These phenomena may influence the effective viscosity and are hard to isolate in physical experiments.

The key features leading to the decrease of viscosity are: 1) self-propulsion, 2) the elongated shape of the swimmers, and 3) the swimmer-swimmer interactions.

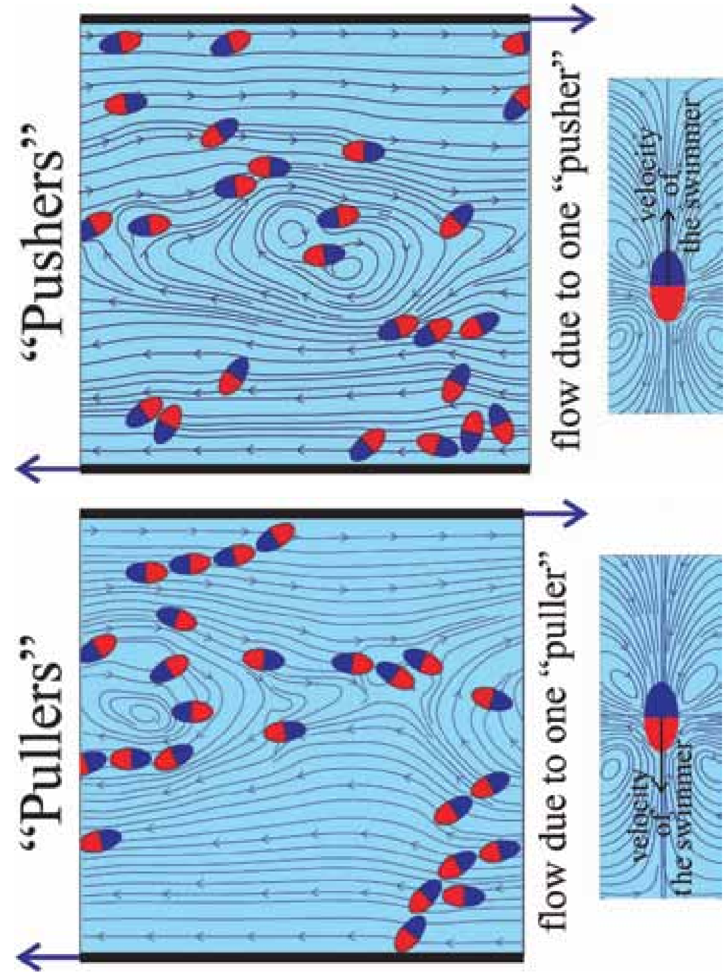

Fig. 1. Snapshots of micro-swimmers in the shear background flow. The blue half of the ellipse represents a solid surface, while the red half represents the surface covered by flagella. The top figure shows pushers (fp > 0), which tend to swim side-by-side. The bottom figure shows pullers (fp < 0), which tend to follow each other forming head-to-tail train-like structures.

The elongated body of the swimmer is modeled by an ellipse. The self-propulsion is modeled by distributing a force (modeling the action of the flagella) of magnitude *fp* over the back half of the ellipse. This force pushes fluid backward (away from the center of the swimmer), resulting in the forward propulsion of the swimmer. The other (front) half of the ellipse represents a solid surface (head of the bacterium). The numerical analysis and experimental observations show that it is important to distinguish between two types of swimmers: pushers (*fp* > 0) – swimmers propelling themselves forward (e.g., *B. subtilis*), and pullers (*fp* < 0) – swimmers propelling themselves backwards (e.g., some algae). Pushers tend to swim side-by-side, while pullers tend to follow each other, forming train-like structures (Fig.1).

The key step in the numerical analysis of the problem was the solution of the incompressible Stokes problem in the fluid domain with complex geometry and mixed boundary conditions. The solution was achieved using the mimetic finite difference (MFD) method [3]. The MFD method combines the mesh flexibility of the finite volume methods with the analytical power of finite element (FE) methods. The MFD method can be viewed as an extension of FE methods to unstructured polygonal meshes. The mesh flexibility simplifies mesh generation around swimmers that may have complicated shapes, e.g., the MFD method allows the mesh elements to have curved edges. Compared with FE methods, the MFD method minimizes the number of discrete unknowns (without loss of accuracy) by: 1) partitioning of the computation domain into a smaller number of elements that are polygons, and 2) using a smaller number of velocity and pressure unknowns only where they are needed for accuracy and stability of the discretization. The MFD method is second-order accurate (with respect to the local mesh size) for the velocity and first-order accurate for the pressure.

Numerically, the effective shear viscosity was measured by placing a suspension of micro-swimmers between two horizontal plates and forcing these plates to move in opposite directions with a constant velocity. The effective viscosity was defined as a coefficient of proportionality between the velocity of the plates and the force required to keep the plates moving.

The numerical simulations showed that the effective shear (Fig.2.) viscosity decays linearly as a function of the propulsion strength *fp*, i.e., pushers decrease the effective viscosity. The linear trend continues for negative values of *fp*, i.e., pullers increase the effective viscosity. For large values of the propulsion strength, the linear trend seems to change. We explain this by the finite size of the fluid domain in our simulations and the dynamics of a single swimmer in a shear flow.

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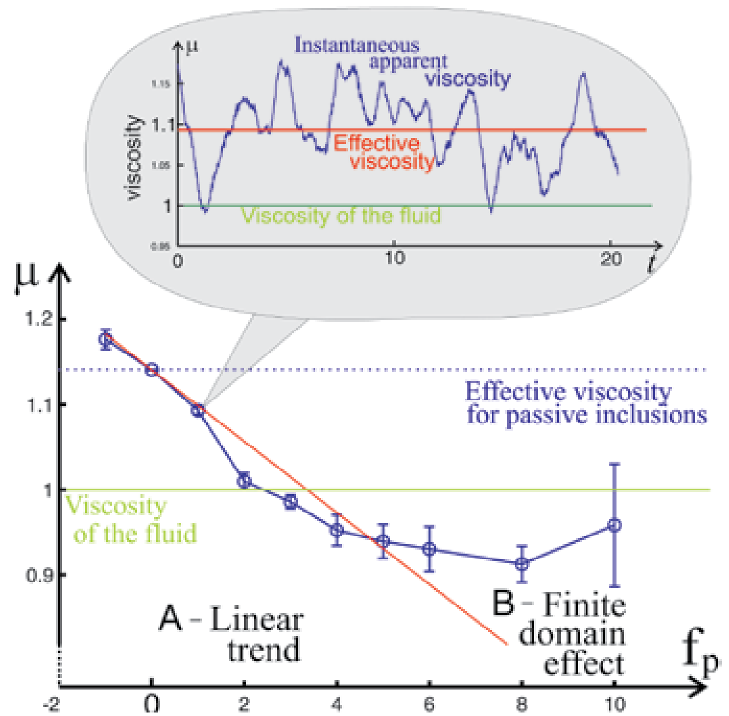

Fig. 2. Suspension of 25 swimmers at moderate concentration (9% by volume) in a unit square with shear velocity boundary conditions on top and bottom sides, and periodic boundary conditions on vertical sides. The green horizontal line indicates the viscosity of the ambient fluid. The blue oscillating line in the top picture shows instantaneous measurements of the shear viscosity. The red horizontal line indicates the time average of the instantaneous measurements. Linear reduction of the effective shear viscosity is observed when propulsion force fp is less than 5. Wall effects become important for larger values of fp.

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[2] V. Gyrya et al., LAUR 09-06018; (2009); submitted to *J. Math. Bio.*
[3] L. Beirão da Veiga et al., *J. Comp. Phys.* **228,** 7215 (2009).

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